Math 3110 | # Test #1 | **September 23^{rd}, 2011**

Name: ____________________________________ **Be sure to show your work!**

1. (____/20 points) Definition and Basics

(a) Suppose that G is a non-empty set equipped an operation. What 4 things do I need to check to see if G is a group?

1:

2:

3:

4:

What additional property needs to hold for G to be an **Abelain** group?

5:

(b) Let $G = \mathbb{R}_{\neq 0}$ be the non-zero real numbers. Also, for each $x, y \in G$ let $x \div y = \dfrac{x}{y}$. Prove G (with this operation) is **not** a group.

(c) Let $G = \{\dots, -16, -9, -4, -1, 0, 1, 4, 9, 16, \dots\}$ (the set of perfect squares and their negatives). Prove G fails to be a group under the operation of addition.

2. (_____/20 points) The group $\mathbb{Z}_n$.

(a) What is the inverse of 4 in $\mathbb{Z}_{12}$?

(b) List all of the cyclic subgroups, $\langle x \rangle$, of $\mathbb{Z}_6$.

(c) What is the order of 8 in $\mathbb{Z}_{12}$?

(d) Draw the subgroup lattice of $\mathbb{Z}_{50}$ [Note: $50 = 2 \cdot 5^2$].

3. (____/23 points) More Modular Arithmetic.

(a) Draw a Cayley table for $U(10)$.

(b) Find $\langle 3 \rangle$ in $U(10)$.

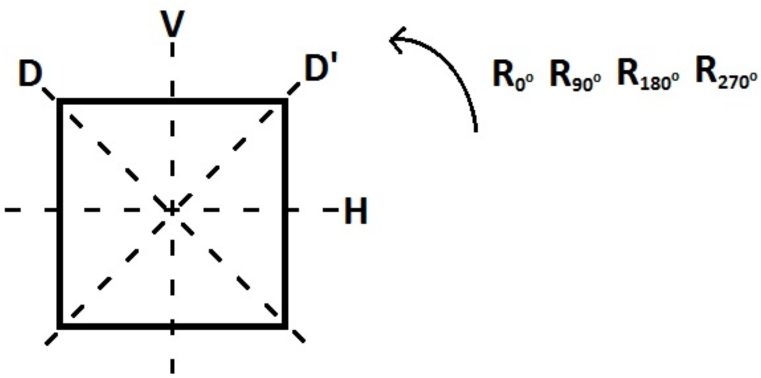
(c) Is $U(10)$ cyclic?

(d) Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$. Is $A \in \mathrm{GL}_2(\mathbb{Z}_{10})$? If so, find A^{-1}. If not, explain why not.

(e) Does 33^{-1} exist in $\mathbb{Z}_{300}$? If so find 33^{-1}. Otherwise explain why it does not exist.

(f) Does 59^{-1} exist in $\mathbb{Z}_{120}$? If so find 59^{-1}. Otherwise explain why it does not exist.

4. (____/15 points) Dihedral Groups.



(a) Compute $D'V$. Draw a few squares to illustrate your computation.

(b) Complete the Cayley table for D_4:

D_4	R_0	R_{90}	R_{180}	R_{270}	H	D'	V	D
R_0	R_0	R_{90}	R_{180}	R_{270}	H	D'	V	D
R_{90}	R_{90}				D'	V		
R_{180}	R_{180}							
R_{270}	R_{270}				D			
H	H	D						R_{90}
D'	D'							
V	V			D	R_{180}			
D	D		D'	H				

(c) In a sentence or two explain why the set of rotations is a subgroup of D_n, but the set of reflections is not.

5. (____/22 points) Proofs!

(a) Let G be a group and suppose that for all $a, b \in G$ we have $(ab)^{-1} = a^{-1}b^{-1}$. Prove G is Abelian.

(b) Let $g \in G$ where G is a group. Prove $\langle g \rangle = \{g^k \mid k \in \mathbb{Z}\}$ is a subgroup of G. Then show $\langle g \rangle$ is Abelian.

(c) Show $H = \{0, 3, 6, 9\}$ is a subgroup of $\mathbb{Z}_{12}$.