Math 3110 **Test #3** **November 18^{th}, 2011**

Name: ______________________________ **Be sure to show your work!**

1. (____/25 points) 3-2-1...Go!

(a) How many elements of order 3 are in $\mathbb{Z}_{9000} \oplus \mathbb{Z}_{3333333}$?

(b) Let H be a subgroup of G where G is abelian. Quickly explain why H is a normal subgroup of G.

(c) Consider for example: $\mathbb{Z}_{12}/\langle 4 \rangle \cong \mathbb{Z}_4$. Now let $k\ell = n$ (where k, ℓ, n are positive integers) and let $G = \mathbb{Z}_n/\langle k \rangle$. Briefly, explain why the G has order k, why G is cyclic, and why $G \cong \mathbb{Z}_k$.

(d) Let G and H be group. Prove that $\pi : G \oplus H \to G$ defined by $\pi((g, h)) = g$ is a homomorphism which is onto. Find the kernel of π. Finally, show that $G \oplus H/\{e\} \oplus H \cong G$.

2. (_____/25 points) Quotients

(a) Given: $K = \{R_0, R_{180}\}$ is a normal subgroup of D_6.

The order of D_6 / K is ______________________________.

The identity of D_6 / K is _______________. $(R_{60}K)^{-1} =$ ______________________________.

The order of $R_{60}K$ in D_6 / K is _______________.   The size of the set $R_{60}K$ is _______________.

Scratch work:

(b) Let H be a (normal) subgroup of G where G is abelian. Prove that G / H is abelian.

(c) Consider $\mathbb{Z}_{12} / H$ where $H = \langle 4 \rangle = \{0, 4, 8\}$. List all of the cosets (and their contents). Then make a Cayley table for this quotient group.

3. (_____/25 points) Oh no! I've made a mistake. $2^2 \cdot 5 \neq 50$

In each of the following situations, explain why we know **a mistake has been made**.
(Why are these statements wrong?)

(a) We found a homomorphism $\varphi : \mathbb{Z}_{50} \oplus \mathbb{Z}_{10} \to D_5$ which is onto.

(b) Let $H = Z(D_6) = \{R_0, R_{180}\}$ (the center of D_6). After some shoddy computations, I've found that $D_6/H \cong \mathbb{Z}_6$. [*Hint:* No computations needed to shoot this down.]

(c) I just found a normal subgroup $H \lhd S_4$ such that S_4/H is an abelian group of order 18.

(d) My friend Herbert found an element $(x, y) \in D_6 \oplus \mathbb{Z}_{10}$ whose order is 60.

4. (_____/25 points) Finite Abelian Groups

(a) List all of the non-isomorphic abelian groups of order $100 = 2^2 5^2$. (*Note*: $4 \cdot 25 = 100 \leftarrow$ I can multiply!)

(b) How many non-isomorphic abelian groups of order 449,878,000 are there?

Note: $449{,}878{,}000 = 2^4 5^3 11^3 13^2$ and there are 5 non-isomorphic abelian groups of order $2^4 = 16$. ☺

(c) Are the groups $\mathbb{Z}_{18} \oplus \mathbb{Z}_{12} \oplus \mathbb{Z}_{25}$ and $\mathbb{Z}_{180} \oplus \mathbb{Z}_{30}$ isomorphic? Explain your answer.

(d) Is the group $\mathbb{Z}_{15} \oplus \mathbb{Z}_{7} \oplus \mathbb{Z}_{121}$ cyclic? (Note: $121 = 11^2$) Explain you answer.