Math 3110 | # Test #2 | **October 9^{th}, 2015**

Name: ______________________________ **Be sure to show your work!**

1. (20 points) Random Group Stuff — Fill out the following table:

$G =$	What is the identity of G?	Is G abelian?	Is G cyclic?	What is the order of ...?	Does G have an element of order 5?
$\mathbb{Z}_{99}$				$\|15\| =$	
$U(10)$				$\|3\| =$	
D_{20}				$\|x^6\| =$	
S_9				$\|(123)(4567)(89)\| =$	

Recall: $D_{20} = \{1, x, \ldots, x^{19}, y, xy, \ldots, x^{19}y\}$ where $x^{20} = 1$, $y^2 = 1$, and $xyxy = 1$.

Scratch Work:

2. (24 points) Cyclic Stuff

(a) Let G be a finite group and $g \in G$. Suppose that $|g| = 66$.

i. What is the order of g^{55}? List the distinct elements in $\langle g^{55} \rangle$.

$$\langle g^{55} \rangle = \left\{ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \right\}$$

ii. Is $g^{22} \in \langle g^{22} \rangle$? **Yes** / **No**

(b) How many elements of order 6 does $\mathbb{Z}_{120}$ have? What are they?

(c) List the orders of elements in $\mathbb{Z}_{55}$. Then determine the number of elements of each order.

Order =						
Number of elements =						

(d) List the orders of elements in D_{55}. Then determine the number of elements of each order.

Order =						
Number of elements =						

3. (22 points) Permutations

(a) Let $G = \langle i \rangle = \{1, i, -1, -i\}$ where $i = \sqrt{-1}$. [G is a subgroup of $\mathbb{C}_{\neq 0}$ (nonzero complex numbers).]
Label 1 as 1, i as 2, -1 as 3, and $-i$ as 4. Cayley's theorem says that G is isomorphic to a subgroup of S_4. Find this subgroup [using left multiplication maps and the labels provided].

$$G \cong \left\{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \right\}$$

(b) Write $\sigma = (237)(1724)(27563)$ as a product of disjoint cycles.

$\sigma^{-1} =$

The order of σ is $|\sigma| =$ ______________.

Write σ as a product of transpositions. σ is **Even** / **Odd**

Compute σ^{30}.

4. (18 points) Explain why the following pairs of groups are not isomorphic.

(a) $\mathbb{Q} \ncong \mathbb{Z}_{123}$ [$\mathbb{Q}$ is the rational numbers.]

(b) $Q \ncong \mathrm{Aut}(\mathbb{Z}_8)$ [$Q = \{\pm 1, \pm i, \pm j, \pm k\}$ is the group of quaternions.]

(c) $A_4 \ncong D_6$

5. (16 points) A few proofs

(a) Explain why $A_3 \cong \mathbb{Z}_3$ but $A_n \ncong \mathbb{Z}_n$ for $n > 3$.

(b) Pick **ONE** of the following...

I. Let G be an abelian group. Show that $\varphi : G \to G$ defined by $\varphi(x) = x^{-1}$ is an automorphism of G.

II. Let φ, ψ be automorphisms of G. Prove that $H = \{x \in G \mid \varphi(x) = \psi(x)\}$ is a subgroup of G.