Math 3110 # Test #2 **October 9^{th}, 2020**

Name: ______________________________ **Be sure to show your work!**

1. (20 points) Random Group Stuff — Fill out the following table:

$G =$	What is the identity of G?	What is the order of ...?	Does G have an element of order 5?	Is G abelian?	Is G cyclic?
$\mathbb{Z}_{80}$		$\|25\| =$			
$U(9)$		$\|2\| =$			
D_{12}		$\|x^8\| =$			
S_9		$\|(1234)(56)(789)\| =$			

Recall: $D_{12} = \{1, x, \ldots, x^{11}, y, xy, \ldots, x^{11}y\}$ where $x^{12} = 1$, $y^2 = 1$, and $xyxy = 1$.

Scratch Work:

2. (20 points) Cyclic Stuff

(a) Let G be a finite group and $g \in G$. Suppose that $|g| = 49$.

i. What is the order of g^{28}? List the distinct elements in $\langle g^{28} \rangle$.

$$\langle g^{28} \rangle = \left\{ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \right\}$$

ii. Is $g^6 \in \langle g^{14} \rangle$? **Yes** / **No**

(b) How many elements of order 8 does $\mathbb{Z}_{40}$ have? What are they?

(c) List the orders of elements in $\mathbb{Z}_{77}$. Then determine the number of elements of each order.

Order =					
Number of elements =					

(d) List the orders of elements in D_{77}. Then determine the number of elements of each order.

Order =					
Number of elements =					

3. (22 points) Permutations *Note:* Please giive simplified ("good manners") answers.

(a) Consider $D_4 = \langle x, y \mid x^4 = 1, y^2 = 1, (xy)^2 = 1 \rangle = \{1, x, x^2, x^3, y, xy, x^2y, x^3y\}$.

Label these elements in the order listed above: 1 as 1, x as 2,... x^3y as 8. Cayley's theorem says that D_4 is isomorphic to a subgroup of S_8. Using left multiplication maps and the labels provided, what element of S_8 represents y?

Assuming that this correspondence associates xy with (16)(25)(38)(47) and x with (1234)(5678). What permuation is associated with x^2y?

(b) Write $\sigma = (1235)(14)(236)$ as a product of disjoint cycles.

$\sigma^{-1} =$

The order of σ is $|\sigma| =$ ______________.

Write σ as a product of transpositions. σ is **Even** / **Odd**

Compute σ^{102}.

4. (18 points) Explain why the following pairs of groups are not isomorphic.

(a) $\mathbb{R}^{2\times 2} \not\cong \mathrm{GL}_2(\mathbb{R})$ [$\mathbb{R}^{2\times 2}$ is the 2×2 matrices under addition.]

(b) $D_4 \not\cong Q$ [$Q = \{\pm 1, \pm i, \pm j, \pm k\}$ is the group of quaternions.]

(c) $A_5 \not\cong \mathbb{Z}_{60}$

5. (20 points) A few proofs

(a) Let $G = \{1, i, -1, -i\}$ where $i = \sqrt{-1}$. [G is a subgroup of $\mathbb{C}_{\neq 0}$ (nonzero complex numbers).]
Prove that $G \cong U(10)$.

(b) We know A_n is not abelian for $n > 3$. Show this.

(c) Pick **ONE** of the following...

I. Define $\varphi : \mathbb{Z}_9 \to \mathbb{Z}_9$ by $\varphi(x) = 4x$. Show φ is an automorphism of $\mathbb{Z}_9$. What is its inverse? Why is $\psi : \mathbb{Z}_8 \to \mathbb{Z}_8$ defined by $\psi(x) = 4x$ **not** an automorphism of $\mathbb{Z}_8$?

II. Show that if G is cyclic, then it must also be abelian. Is the converse also true? Why or why not? Justify your answer(s).