

Name: _____

Don't merely state answers, prove your statements. **Be sure to show your work!**

The last page is a copy of Cayley tables for D_3 and D_4 .

The tables should be helpful when doing problems 3 and 4.

1. (____/20 points) Isomorphic or not?

(a) Is $GL_2(\mathbb{R}) \cong \mathbb{Z}_{100}$? Why or why not?

(b) Recall $A_4 = \{(1), (123), (132), (124), (142), (134), (143), (234), (243), (12)(34), (13)(24), (14)(23)\}$. Show $A_4 \not\cong D_6$ (they are not isomorphic). *Hint:* Think about orders of elements.

(c) G is a group of order 5. Is $G \cong \mathbb{Z}_5$? Yes, no, maybe? Explain your answer.

2. (____/20 points) Homomorphisms

- (a) Let $G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \mid a \in \mathbb{R} \right\}$ and let $\varphi : G \rightarrow \mathbb{R}$ be defined by $\varphi \left(\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \right) = a$. Show that φ is a homomorphism and find the kernel of φ . Is φ 1-1? onto? an isomorphism?

- (b) Let $\psi : \mathbb{Z}_9 \rightarrow \mathbb{Z}_3$ be defined by $\psi(n) = n$. Is ψ a *well defined* homomorphism? If so, find $\text{Ker}(\psi)$. Is ψ 1-1? onto? an isomorphism?

3. (____/20 points) Write down the left multiplication operators for D_3 and translate them into permutations (in S_6) labeling the elements of D_3 as follows: $1 \rightsquigarrow 1$, $a \rightsquigarrow 2$, $a^2 \rightsquigarrow 3$, $b \rightsquigarrow 4$, $ab \rightsquigarrow 5$, $a^2b \rightsquigarrow 6$. Write down a subgroup of S_6 which isomorphic to D_3 (using Cayley's theorem).

4. (____/20 points) Let $H = \{1, x^2\}$ (H is a subgroup of D_4).

(a) Find all of the left and right cosets of H in D_4 . Is H a normal subgroup of D_4 ?

(b) Write out a Cayley table for the quotient group D_4/H .

5. (____/20 points) Let $\varphi : G \rightarrow G'$ be a homomorphism.

(a) Prove that $\text{Ker}(\varphi)$ is a subgroup of G . Then show that $\text{Ker}(\varphi)$ is a **normal** subgroup of G .

(b) Show that $\varphi(G)$ is a subgroup of G' .

D_3	1	a	a^2	b	ab	a^2b
1	1	a	a^2	b	ab	a^2b
a	a	a^2	1	ab	a^2b	b
a^2	a^2	1	a	a^2b	b	ab
b	b	a^2b	ab	1	a^2	a
ab	ab	b	a^2b	a	1	a^2
a^2b	a^2b	ab	b	a^2	a	1

D_4	1	x	x^2	x^3	y	xy	x^2y	x^3y
1	1	x	x^2	x^3	y	xy	x^2y	x^3y
x	x	x^2	x^3	1	xy	x^2y	x^3y	y
x^2	x^2	x^3	1	x	x^2y	x^3y	y	xy
x^3	x^3	1	x	x^2	x^3y	y	xy	x^2y
y	y	x^3y	x^2y	xy	1	x^3	x^2	x
xy	xy	y	x^3y	x^2y	x	1	x^3	x^2
x^2y	x^2y	xy	y	x^3y	x^2	x	1	x^3
x^3y	x^3y	x^2y	xy	y	x^3	x^2	x	1