Math 3110 # Test #1 **February 22^{nd}, 2010**

Name: ________________________________

Don't merely state answers, prove your statements. **Be sure to show your work!**

1. (____/10 points) Let $S = \mathbb{R} - \{0\}$ (the set of non-zero reals). For $x, y \in S$, define $x \star y = x^{-1}y$. [Example: $5 \star 10 = 10/5 = 2$]

(a) Is S closed with respect to $\star$?

(b) The operation $\star$ is **not** associative. Give a example which shows this.

2. (____/15 points) Here are 3 collections of subsets of $\mathbb{Z}$. 2 of these collections are **not** partitions – **explain why they fail to be partitions**. 1 of these collections is a partition – **describe the corresponding equivalence relation**.

(a) $\{-1, -2, -3, \dots\}$ and $\{1^2, 2^2, 3^2, 4^2, \dots\}$

(b) $\{\dots, -6, -3, 0, 3, 6, 9, \dots\} = \{3k \mid k \in \mathbb{Z}\}$,
$\{\dots, -5, -2, 1, 4, 7, \dots\} = \{3k+1 \mid k \in \mathbb{Z}\}$, and
$\{\dots, -4, -1, 2, 5, 8, \dots\} = \{3k+2 \mid k \in \mathbb{Z}\}$.

(c) $\{\dots, -3, -2, -1, 0\}$ and $\{0, 1, 2, 3, \dots\}$

3. (____/15 points) One-to-one and onto.

(a) Let $f : A \to B$ and $g : B \to A$. Suppose that $g \circ f$ is bijective. Prove **ONE** of the following:

i. f is injective (i.e. one-to-one).

ii. g is surjective (i.e. onto).

(b) Let $h : \mathbb{Z} \to \mathbb{Z}$ be defined by $h(x) = 2x$. Is h one-to-one? Is h onto? **Prove your answers.**

4. (____/20 points) Quick Proofs

(a) Using induction, show that $n < 2^n$ for all **non-negative** integers n. *Hint:* $1 + 2^n \leq 2^n + 2^n$.
YOU MUST USE INDUCTION!

(b) Let $f : A \to B$ and $S_1, S_2 \subseteq A$. Show that $f(S_1) \cup f(S_2) \subseteq f(S_1 \cup S_2)$.

5. (____/20 points) Divisibility

(a) Use the Euclidean algorithm to find $(4, 11) = d$ (i.e. GCD of 4 and 11). Then use your work to backtrack through the algorithm and find integers x and y such that $4x + 11y = d$

(b) Suppose $ax + by = 6$ for some integers a, b, x, y. What are the possible value(s) of (a, b)?

(c) Let $a, b, c \in \mathbb{Z}$ such that a and b are relatively prime, $a \Big| c$, and $b \Big| c$. Show that $ab \Big| c$.

6. (____/20 points) Workin' mod 6

(a) Finish filling out the following addition and multiplication tables for $\mathbb{Z}_6$ (operations are addition and multiplication "mod 6"):

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1					
2	2					
3	3					
4	4					
5	5					

$\times$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2				
3	0	3				
4	0	4				
5	0	5				

(b) For each $x \in \mathbb{Z}_6$, either find x^{-1} or write "DNE" (if the multiplicative inverse does not exist).

$0^{-1} =$	$1^{-1} =$	$2^{-1} =$	$3^{-1} =$	$4^{-1} =$	$5^{-1} =$

(c) For each $x \in \mathbb{Z}_6$, either find $-x$ or write "DNE" (if the additive inverse does not exist).

$-0 =$	$-1 =$	$-2 =$	$-3 =$	$-4 =$	$-5 =$

(d) Compute $2^{-1}(5-1)+3$ or explain why this is undefined.

(e) Compute $5^{-1}(3+4)-2$ or explain why this is undefined.