

Name: _____

Be sure to show your work!

1. (20 points) Random Group Stuff — Fill out the following table:

$G =$	What is the identity of G ?	Is G abelian?	Is G cyclic?	What is the order of ...?	Does G have an element of order 10?
\mathbb{Z}_{100}				$ 15 =$	
$U(4)$				$ 3 =$	
D_5				$ x^2y =$	
S_7				$ {(12)(3456)} =$	

Recall: $D_5 = \{1, x, \dots, x^4, y, xy, \dots, x^4y\}$ where $x^5 = 1$, $y^2 = 1$, and $xyxy = 1$.

Scratch Work:

2. (24 points) Cyclic Stuff

(a) Let G be a finite group and $g \in G$. Suppose that $|g| = 35$.

i. What is the order of g^{10} ? List the distinct elements in $\langle g^{10} \rangle$.

$$\langle g^{10} \rangle = \left\{ \qquad \qquad \qquad \right\}$$

ii. Is $g^{100} \in \langle g^{10} \rangle$? **Yes** / **No**

(b) How many elements of order 8 does \mathbb{Z}_{40} have? What are they?

(c) List the orders of elements in \mathbb{Z}_{35} . Then determine the number of elements of each order.

Order =							
Number of elements =							

(d) List the orders of elements in D_{35} . Then determine the number of elements of each order.

Order =							
Number of elements =							

3. (22 points) Permutations

Recall that $Q = \{1, -1, i, -i, j, -j, k, -k\}$ where $ij = k, jk = i, ki = j, ji = -k, i^2 = j^2 = k^2 = -1$, etc.

- (a) Write down what the left multiplication operator of i does in Q . Then write down the corresponding permutation if we label 1 as 1, -1 as 2, i as 3, $-i$ as 4, j as 5, $-j$ as 6, k as 7, and $-k$ as 8.

$$L_i : Q \rightarrow Q$$

$$\begin{array}{l} 1 \mapsto \underline{\hspace{2cm}} \\ -1 \mapsto \underline{\hspace{2cm}} \\ i \mapsto \underline{\hspace{2cm}} \\ -i \mapsto \underline{\hspace{2cm}} \\ j \mapsto \underline{\hspace{2cm}} \\ -j \mapsto \underline{\hspace{2cm}} \\ k \mapsto \underline{\hspace{2cm}} \\ -k \mapsto \underline{\hspace{2cm}} \end{array}$$

The corresponding permutation is...?

- (b) Suppose that using Cayley's theorem we found the left multiplication operator of x in D_3 corresponds to $(123)(456)$ and y corresponds to $(14)(26)(35)$. What would the left multiplication operator of x^2y correspond to? [Your answer should be a permutation written as a product of disjoint cycles.]

- (c) Write $\sigma = (152)(3467)(25)(37)$ as a product of disjoint cycles.

$$\sigma^{-1} =$$

The order of σ is $|\sigma| = \underline{\hspace{2cm}}$.

Write σ as a product of transpositions. σ is **Even** / **Odd**

Compute σ^{65} .

4. (18 points) Explain why the following pairs of groups are not isomorphic.

(a) $U(1000) \not\cong \mathbb{R}$

(b) $A_5 \not\cong \mathbb{Z}_{60}$

(c) $S_4 \not\cong D_{12}$

5. (16 points) A few proofs

(a) Prove that $U(10) \cong \mathbb{Z}_4$.

(b) Pick **ONE** of the following...

- I. Let $g \in G$ (G a group). Show that $\varphi : G \rightarrow G$ defined by $\varphi(x) = gxg^{-1}$ is an automorphism of G .
- II. Show that if G is a cyclic group, then G is abelian. Is the converse true?