Math 3110 # Test #2 **March 12^{th}, 2021**

Name: ________________________________ **Be sure to show your work!**

1. (20 points) Random Group Stuff — Fill out the following table:

$G =$	What is the identity of G?	What is the order of ...?	Does G have an element of order 4?	Is G abelian?	Is G cyclic?
$\mathbb{Z}_{100}$		$\|55\| =$			
$U(12)$		$\|5\| =$			
D_{15}		$\|x^9\| =$			
S_8		$\|(12345)(678)\| =$			

Recall: $D_{15} = \{1, x, \ldots, x^{14}, y, xy, \ldots, x^{14}y\}$ where $x^{15} = 1$, $y^2 = 1$, and $xyxy = 1$.

Scratch Work:

2. (20 points) Cyclic Stuff

(a) Let G be a finite group and $g \in G$. Suppose that $|g| = 63 = 3^2 \cdot 7$.

i. What is the order of g^{45}? List the distinct elements in $\langle g^{45} \rangle$.

$$\langle g^{45} \rangle = \left\{ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \right\}$$

ii. Is $g^{13} \in \langle g^{55} \rangle$? **Yes** / **No**

(b) How many elements of order 12 does $\mathbb{Z}_{36}$ have? What are they?

(c) List the orders of elements in $\mathbb{Z}_{35}$. Then determine the number of elements of each order.

Order =					
Number of elements =					

(d) List the orders of elements in D_{35}. Then determine the number of elements of each order.

Order =					
Number of elements =					

3. (22 points) Permutations *Note:* Please giive simplified ("good manners") answers.

(a) Consider $G = \langle i \rangle = \{1, i, -1, -i\}$ where $i = \sqrt{-1}$ so that $i^2 = -1$. Label 1 as 1, i as 2, -1 as 3, and $-i$ as 4. Cayley's theorem says that G is isomorphic to a subgroup of S_4. Find this subgroup [using left multiplication maps and the labels provided].

$$G \cong \left\{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \right\}$$

(b) Write $\sigma = (243)(1523)(364)$ as a product of disjoint cycles.

$\sigma^{-1} =$

The order of σ is $|\sigma| =$ ______________.

Write σ as a product of transpositions. σ is **Even** / **Odd**

Compute σ^{64}.

(c) Does S_{10} have an element of order 30? If not, explain why not. If so, give an example of such an element.

4. (18 points) Explain why the following pairs of groups are not isomorphic.

(a) $\mathbb{Z}_{12} \ncong U(12)$

(b) $S_4 \ncong D_{12}$

(c) $A_5 \ncong \mathbb{Z}_{60}$

5. (20 points) A few proofs

(a) Explain why $S_2 \cong \mathbb{Z}_2$ but $S_n \ncong \mathbb{Z}_{n!}$ for any $n > 2$.

(b) Pick **ONE** of the following...

I. Let G be a group and $g \in G$. Show that $\varphi : G \to G$ defined by $\varphi(x) = gxg^{-1}$ is an automorphism.

II. Let $\psi : G_1 \to G_2$ be an isomorphism and suppose G_1 is Abelian. Show G_2 is Abelian.