Math 3220 # Test #1 **February** 13^{th}**, 2019**

Name: ______________________________ **Be sure to show your work!**

1. (12 points) Definitions

(a) Give the definition of a *Cauchy* sequence for a sequence $\{x_n\}_{n=1}^{\infty}$.

(b) Let $X \subseteq \mathbb{R}$ be a non-empty set. Give the definition of $\inf(X)$.

2. (25 points) Determine which of the following statements are true or false. Circle your answer.

(a) **True** / **False** $\mathbb{Q}$ (the rational numbers) and $\mathbb{R} - \mathbb{Q}$ (the irrational numbers) have the same cardinality.

(b) **True** / **False** The set $A = \{r \in \mathbb{Q} \mid 0 < r < \sqrt{2}\}$ is countably infinite.

(c) **True** / **False** If $|x - 3| > 1$, then $x > 4$.

(d) **True** / **False** If $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence, then every subsequence of $\{x_n\}_{n=1}^{\infty}$ converges.

(e) **True** / **False** Convergent sequences are bounded.

(f) **True** / **False** Bounded monotone sequences converge.

(g) **True** / **False** If $X \subseteq \mathbb{R}$ is a non-empty set, then we can always conclude that $\inf(X) < \sup(X)$.

(h) **True** / **False** If $\{a_n\}_{n=1}^{\infty}$ converges and $\{b_n\}_{n=1}^{\infty}$ diverges, then $\{a_n + b_n\}_{n=1}^{\infty}$ diverges.

(i) **True** / **False** If $a_n \to A$ and $b_n \to B$, then $a_n/b_n \to A/B$.

(j) **True** / **False** If $\lim\limits_{n\to\infty} a_n = -\infty$, then $\lim\limits_{n\to\infty} 1/a_n = 0$.

3. (14 points) Examples requested.

(a) Give an example of a set $A \subseteq \mathbb{R}$ such that $\inf(A) = -4 \notin A$ and $\sup(A) = \infty$.

(b) Give an example of a *non-monotonic* sequence which converges to 5.

4. (15 points) Let S be a non-empty set of *positive* real numbers. Moreover, assume S is bounded above. First, why does $\alpha = \sup(S)$ exist? Next, show that $\sup(S^2) \leq \alpha^2$ where $S^2 = \{s^2 \mid s \in S\}$.

5. (12 points) List off the first 4 terms in the sequence $\left\{ \frac{(-1)^n n + 1}{n} \right\}_{n=1}^{\infty}$. Then show it diverges.

6. (22 points) Use the *definition of convergence* in the following proofs. Don't appeal to "big theorems".

(a) Show that $\left\{ \frac{2n+3}{n^2+1} \right\}_{n=1}^{\infty}$ converges.

(b) Suppose $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ converge. Show that $\{a_n + b_n\}_{n=1}^{\infty}$ converges.