Math 4710/5710 **In the Midst-term Exam — Take Home** **November 14^{th}, 2014**

Due: Tuesday, November 18^{th}, 2014 at 5pm.

Name: _____ **Be sure to show your work!**

The following are True/Possible/False questions. If the statement is always true, state "TRUE" and then prove the statement. If the statement is never true, state "FALSE" and then prove the statement cannot ever hold. If the state is sometimes true and sometimes false, state "POSSIBLE" and then give an example of it holding and an example of it failing to hold.

Each problem is worth **10 points**. Undergrads should complete **5** out of **7** (you may do them all for potential extra credit). Grad students must complete all of these.

1. Let $\mathcal{T}$ be a topology for the set $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

   Is it TRUE, POSSIBLE, or FALSE that $|\mathcal{T}| = 5$?

2. Let X be a metrizable topological space.

   Is it TRUE, POSSIBLE, or FALSE that X has a non-metrizable subspace?

3. X is a path connected topological space.

   Is it TRUE, POSSIBLE, or FALSE that X is still path connected when given a coarser topology?

4. Let X be a non-empty simply ordered set. Give X the order topology. Let $a, b \in X$ with $a < b$.

   Is it TRUE, POSSIBLE, or FALSE that $\overline{(a, b)} = [a, b]$?

5. Let X be a finite topological space. Let $f : X \to Y$ be continuous.

   Is it TRUE, POSSIBLE, or FALSE that $f(X)$ is compact?

6. Let $f : \mathbb{R} \to \mathbb{R}_\ell$ be a continuous map.

   Is it TRUE, POSSIBLE, or FALSE that $f(0) = 0$ and $f(1) = 1$?

7. Let A be a countable closed subset of $X = [0, 1] \times [0, 1]$ (the unit square in $\mathbb{R}^2$). Let $f : \mathbb{R} \to A$ be continuous.

   Is it TRUE, POSSIBLE, or FALSE that there is an open interval $I = (a, b) \subseteq \mathbb{R}$ such that f restricted to I is constant?