**Math 5230-101** **Big Quiz – Extended Edition** **November 9th, 2016**

Hopefully, not too surprising.

A few parts in red were deleted for sake of time.

**Name:** ________________________________ **Be sure to show your work!**

**1.** Let $W = \left\{ \begin{bmatrix} a+b & a+b \\ a-c & a-c \end{bmatrix} \;\middle|\; a,b,c \in \mathbb{R} \right\}$. Show $W$ is a subspace using the subspace test. Then again show it is a subspace by recognizing it as a span or a kernel. Find a basis for $W$.

**2.** Let $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2\times 2} \;\middle|\; a-b=0 \text{ and } c-d=0 \right\}$. Show $V$ is a subspace using the subspace test. Then again show it is a subspace by recognizing it as a span or a kernel. Find a basis for $V$.

**X.** With $W$ and $V$ defined above. Show $W \subseteq V$. Is $W = V$?

**3.** Let $U = \left\{ \begin{bmatrix} a & a^2 \\ a^2 & a \end{bmatrix} \;\middle|\; a \in \mathbb{R} \right\}$. $U$ is not a subspace. Explain why not.

**4.** Let $\alpha = \{1, x, x^2\}$ (note the order) be the standard basis for $P_2 = \{ax^2 + bx + c \mid a, b, c \in \mathbb{R}\}$. Let $\beta = \{1, 1+x, 1+x+x^2\}$. Show $\beta$ is linearly independent using the definition of linear independence. Find the change of coordinate matrix $[I]_\alpha^\beta$.

Show $\beta$ spans from definition.

**5.** Let $f(x)$ and $g(x)$ be some elements in $P_2$. How do I know for certain that $\delta = \{f(x), g(x)\}$ is not a basis for $P_2$? What can be said? What's possible?

**6.** Let $T : P_2 \to \mathbb{R}^{2\times 2}$ where $T(ax^2+bx+c) = \begin{bmatrix} a & a+b \\ a-b & 0 \end{bmatrix}$. Prove that $T$ is linear. Find a standard coordinate matrix for $T$. Find a basis for $T$'s kernel and range. State $T$'s rank, nullity and whether it is one-to-one, onto, both or neither.

**7.** Suppose $T : V \to W$ is a linear map. Let $\alpha$ and $\gamma$ be bases for $V$, and $\beta$ and $\kappa$ be bases for $W$. Also, suppose that...

$$A = [T]_\alpha^\beta = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \qquad B = [I]_\gamma^\alpha = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad C = [I]_\kappa^\beta = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$\dim(V) =$ ______________ $\dim(W) =$ ______________.

Find $[T]_\gamma^\kappa$.

**8.** Give a quick argument using bases to prove that $\dim(W) \leq \dim(V)$ if $W$ is a subspace of $V$.

**9.** Let $U$ and $W$ be subspaces of some vector space $V$. What does $V = U \oplus W$ mean? (Give a definition.)

**10.** Let $\alpha = \{\mathbf{u}\}$ be a basis for $U$. Let $\beta = \{\mathbf{v}, \mathbf{w}\}$ be a basis for $W$. Assume that $V = U \oplus W$. Prove that $\gamma = \alpha \cup \beta$ is a basis for $V$. [Do not just say, "It's a theorem."]